# Multiple particle-hole pair creation in the harmonically driven Fermi-Hubbard model 

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#### Abstract

We study the Fermi-Hubbard model in the strongly correlated Mott phase under the influence of a harmonically oscillating hopping rate $J(t)=J_{0}+\Delta J \cos (\omega t)$. Apart from the well-known fundamental resonance, where the frequency $\omega$ of this oscillation equals (or a little exceeds) the Mott gap, we also find higher-order resonances where multiple particle-hole pairs are created when $\omega$ is near an integer multiple of the gap. These findings should be relevant for experimental realizations such as ultracold fermionic atoms in optical lattices or pump-probe experiments using laser pulses incident on correlated electrons in solid-state materials.


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## I. INTRODUCTION

Understanding interacting quantum many-body systems is a major challenge in physics-both from the theoretical and the experimental point of view. One reason is the complexity of these systems, as reflected in the dimension of the Hilbert space, which grows exponentially with the particle number. As a result, it can already be very difficult to determine the equilibrium (e.g., ground state) properties of such systems, but it can be even more challenging to understand their dynamics out of equilibrium. Interesting questions in this context are: How do interacting quantum many-body systems react to an external stimulus which drives them out of equilibrium, and how do they relax back?

In this paper, we focus on the first part of that question. As a prototypical example, we consider the Fermi-Hubbard model [1] $(\hbar=1)$

$$
\begin{equation*}
\hat{H}_{\mathrm{FH}}=-J \sum_{\langle\mu, \nu\rangle, s} \hat{c}_{\mu, s}^{\dagger} \hat{c}_{\nu, s}+U \sum_{\mu} \hat{n}_{\mu}^{\uparrow} \hat{n}_{\mu}^{\downarrow} \tag{1}
\end{equation*}
$$

Here $\hat{c}_{\mu, s}^{\dagger}$ and $\hat{c}_{\nu, s}$ are the fermionic creation and annihilation operators at the neighboring lattice sites $\mu$ and $\nu$ with the $\operatorname{spin} s$ while $\hat{n}_{\mu}^{s}$ is the corresponding number operator. Even though it might seem quite simple, this model (1) displays highly nontrivial effects due to the competition between the two (noncommuting) contributions, the hopping rate $J$, and the on-site repulsion $U$. Here, we consider symmetric lattices where all lattice sites $\mu$ and all neighboring pairs $\mu$ and $\nu$ are equivalent.

Depending on lattice structure, temperature, and filling (doping), etc., the Fermi-Hubbard model (1) has a rich phase diagram. We focus on a corner of this phase diagramthe strongly correlated Mott insulator phase at zero temperature and half filling $\left\langle\hat{n}_{\mu}^{\uparrow}\right\rangle=\left\langle\hat{n}_{\mu}^{\downarrow}\right\rangle=1 / 2$. Thus, we assume that the hopping rate $J$ is much smaller than the on-site repulsion $U$. Apart from small quantum corrections (due to "virtual" hopping processes), the ground state has one particle per lattice site as double occupancy costs an energy penalty of $U$. This state is insulating with a Mott gap of approximately $U$, up to corrections of order $\mathcal{O}(J)$ or higher due to these virtual hopping processes which lower the energy.

Starting with the Mott insulator state as the initial ground (equilibrium) state, we study how the Fermi-Hubbard model (1) reacts to an external stimulus $J(t)$. A frequently studied and conceptually clear example is a quantum quench,
which denotes the following sequence: The system starts in the ground (or thermal equilibrium) state and one of the parameters such as the hopping rate $J$ is changed suddenly $J(t)=J_{\text {in }}+\Delta J \Theta(t)$ where $\Theta(t)$ is the Heaviside step function. After that, the system is no longer in its ground or equilibrium state in general. The subsequent real-time evolution including phenomena like decaying oscillations [2] and the different stages of relaxation [3-5] such as prethermalization and thermalization [6-9] and the spreading of correlations [10] have been studied in various works.

In the following, we consider a different stimulus $J(t)$ and assume an oscillating profile

$$
\begin{equation*}
J(t)=J_{0}+\Delta J \cos (\omega t), \tag{2}
\end{equation*}
$$

which is often studied in the context of driven quantum systems, e.g., regarding the creation [11] or directed motion [ 12,13 ] of doublons and holons or new nonequilibrium phases; see also [14]. Depending on the experimental realization, such an oscillating hopping rate could be generated by periodic modulations of external parameters such as pressure, magnetic or electric field, etc. For atoms in optical lattices, such a $J(t)$ can be induced by varying the laser intensity or by shaking the lattice. In pump-probe spectroscopy, $J(t)$ can represent the impact of the pump beam on the investigated material system; see the Appendix.

Since both $J_{0}$ and $\Delta J$ are supposed to be small in comparison to $U$ (deep in the Mott phase), the reaction of the quantum system, i.e., the departure from equilibrium caused by the external stimulus, will be most pronounced for resonant excitation. Naively, one might expect that this excitation mechanism is only effective if the driving frequency $\omega$ equals (approximately) the Mott gap $U+\mathcal{O}(J)$ which corresponds to the energy needed to create one particle-hole pair. While this fundamental resonance is most efficient in general, we also find higher-order resonances where the driving frequency $\omega$ equals (approximately) the energy of two or more pairs.

## II. PAIR CREATION

Let us first study the well-known fundamental resonance. As one way to understand this process, let us employ the method of the hierarchy of correlations. This approach is described in [15-17] and particularly adapted to the treatment of nonequilibrium phenomena in strongly correlated lattice systems. To this end, we consider the reduced density matrix


FIG. 1. Connectivity diagrams of the considered lattice geometries: a tetrahedron as 3D representation (top left) and as planar diagram (top middle); a square (top right); and fully permutationally invariant lattices with six sites (bottom left) and with eight sites as planar diagram (bottom middle) and as 3D representation (bottom right).
$\hat{\varrho}_{\mu}$ of one lattice site $\mu$ and analogously $\hat{\varrho}_{\mu \nu}$ for two lattice sites $\mu$ and $\nu$, etc. Separating the correlated part via $\hat{\varrho}_{\mu \nu}=\hat{\varrho}_{\mu \nu}^{\text {corr }}+\hat{\varrho}_{\mu} \hat{\varrho}_{\nu}$, we may derive the evolution equations for $\partial_{t} \widehat{\varrho}_{\mu \nu}^{\text {corr }}$, etc. To lowest order, the ground state (Mott insulator) restricted to two lattice sites can be represented by the equipartition state $|\uparrow, \downarrow\rangle_{\mu \nu}=\hat{c}_{\mu, \uparrow}^{\dagger} \hat{c}_{\nu, \downarrow}^{\dagger}|0\rangle$ while the state with a doublon-holon excitation at these two sites can be written as $|\uparrow \downarrow, 0\rangle_{\mu \nu}=\hat{c}_{\mu, \uparrow}^{\dagger} \hat{c}_{\mu, \downarrow}^{\dagger}|0\rangle$. Calculating the matrix element of $\hat{\varrho}_{\mu \nu}^{\text {corr }}(t)$ between these two states, we find

$$
\begin{equation*}
\langle\uparrow \downarrow, 0|\left(i \partial_{t}-U\right) \hat{\varrho}_{\mu \nu}^{\mathrm{corr}}(t)|\uparrow, \downarrow\rangle=J(t) \mathcal{M}_{\mu \nu}^{(2)} \tag{3}
\end{equation*}
$$

where $\mathcal{M}_{\mu \nu}^{(2)}$ denotes a matrix element containing the on-site matrices $\hat{\varrho}_{\mu}$ and $\hat{\varrho}_{\nu}$; for example, cf. [15-17]. Evidently, if $J(t)$ oscillates with the frequency $\omega=U+\mathcal{O}(J)$, we would get a resonant growth of $\hat{\varrho}_{\mu \nu}^{\text {corr }}(t)$ corresponding to particle-hole (doublon-holon) pair creation.

## A. Double pair creation

As mentioned above, the well-known fundamental resonance condition $\omega=U+\mathcal{O}(J)$ is not the only possibility. As we demonstrate below, for $\omega=2 U+\mathcal{O}(J)$, one could resonantly create two particle-hole pairs at the same time, for example. This effect can be understood analogously in terms of the four-point correlator $\hat{\varrho}_{\mu \nu \lambda \sigma}^{\text {corr }}$ the matrix element of which obeys the equation

$$
\begin{equation*}
\langle\uparrow \downarrow, 0, \uparrow \downarrow, 0|\left(i \partial_{t}-2 U\right) \hat{\varrho}_{\mu \nu \lambda \sigma}^{\mathrm{corr}}(t)|\uparrow, \downarrow, \uparrow, \downarrow\rangle=J(t) \mathcal{M}_{\mu \nu \lambda \sigma}^{(4)} \tag{4}
\end{equation*}
$$

The remaining matrix element $\mathcal{M}_{\mu \nu \lambda \sigma}^{(4)}$ contains products of two-point correlations such as $\hat{\varrho}_{\mu \nu}^{\text {corr }} \hat{\varrho}_{\lambda \sigma}^{\text {corr }}$. Thus, we necessarily obtain resonant creation of two particle-hole (doublon-holon) pairs at the same time-unless the source term $\mathcal{M}_{\mu \nu \lambda \sigma}^{(4)}$ vanishes identically.

In order to show that this source term is nonvanishing, let us consider a simple and exactly solvable case-the Fermi-Hubbard model (1) on a tetrahedron (see Fig. 1), i.e., two spin-up plus two spin-down fermions on four lattice sites with full permutation invariance. For vanishing hopping
$J=0$, the ground state $\left|\psi_{\text {ground }}\right\rangle$ is the fully symmetrized state $\left|\psi_{\text {ground }}\right\rangle=|\uparrow, \downarrow, \uparrow, \downarrow\rangle_{\text {symm }}$ which we shall denote by $\left|\phi_{0}\right\rangle=|\uparrow, \downarrow, \uparrow, \downarrow\rangle_{\text {symm }}$. Analogously, the first excited state reads $\left|\psi_{\text {first }}\right\rangle=|\uparrow \downarrow, 0, \uparrow, \downarrow\rangle_{\text {symm }}$ which will be abbreviated by $\left|\phi_{1}\right\rangle=|\uparrow \downarrow, 0, \uparrow, \downarrow\rangle_{\text {symm }}$. Finally, the second excited (i.e., highest-energy) state is $\left|\psi_{\text {second }}\right\rangle=|\uparrow \downarrow, 0, \uparrow \downarrow, 0\rangle_{\text {symm }}$ and will be denoted by $\left|\phi_{2}\right\rangle=|\uparrow \downarrow, 0, \uparrow \downarrow, 0\rangle_{\text {symm }}$.

In this case $J=0$, the matrix element $\left\langle\phi_{2}\right| \hat{H}_{\Delta J}\left|\phi_{0}\right\rangle$ between the lowest- and highest-energy state would be zero since one cannot go from $\left|\phi_{0}\right\rangle$ to $\left|\phi_{2}\right\rangle$ with only one hopping event. For small $J>0$, however, the ground state $\left|\psi_{\text {ground }}\right\rangle$ also contains a small $\mathcal{O}(J)$ admixture of $\left|\phi_{1}\right\rangle$ and an even smaller $\mathcal{O}\left(J^{2}\right)$ of $\left|\phi_{2}\right\rangle$. As one way to see this, one can exactly diagonalize the Hamiltonian (1) for this simple case. Using the three vectors $\left|\phi_{0}\right\rangle,\left|\phi_{1}\right\rangle$, and $\left|\phi_{2}\right\rangle$ mentioned above as a basis for the fully permutation-invariant subspace of the Hilbert space, the Hamiltonian (1) can be represented by a $3 \times 3$ matrix of the following form:

$$
\hat{H}_{\mathrm{FH}}=\left(\begin{array}{ccc}
0 & -4 J & 0  \tag{5}\\
-4 J & U-4 J & -4 J \\
0 & -4 J & 2 U
\end{array}\right)
$$

Diagonalization of this matrix yields the ground state (for small but nonzero values of $J$ )

$$
\begin{align*}
\left|\psi_{\text {ground }}\right\rangle= & \left(1-8 \frac{J^{2}}{U^{2}}\right)\left|\phi_{0}\right\rangle+4\left(\frac{J}{U}+4 \frac{J^{2}}{U^{2}}\right)\left|\phi_{1}\right\rangle \\
& +8 \frac{J^{2}}{U^{2}}\left|\phi_{2}\right\rangle+\mathcal{O}\left(\frac{J^{3}}{U^{3}}\right) \tag{6}
\end{align*}
$$

E.g., if we suddenly switched off $J$ (quantum quench), this admixture of $\left|\phi_{1}\right\rangle$ or $\left|\phi_{2}\right\rangle$ contained in $\left|\psi_{\text {ground }}\right\rangle$ would then yield the amplitude for creating one or two pairs by this quantum quench. Analogous expressions can be derived for the first and second excited state $\left|\psi_{\text {first }}\right\rangle$ and $\left|\psi_{\text {second }}\right\rangle$ containing one and two particle-hole pairs, respectively. Now, splitting the total time-dependent Hamiltonian (1) into an unperturbed stationary part $\hat{H}_{0}$ and a time-dependent perturbation $\hat{H}_{\Delta J}$ via $\hat{H}_{\mathrm{FH}}(t)=\hat{H}_{0}+\hat{H}_{\Delta J}$ and calculating the matrix element of the perturbation Hamiltonian $\hat{H}_{\Delta J}$ between the ground state and the highest-energy state-which corresponds to the resonant generation of two pairs at the same time-we find that these admixtures yield a nonzero amplitude:

$$
\begin{equation*}
\left\langle\psi_{\text {second }}\right| \hat{H}_{\Delta J}\left|\psi_{\text {ground }}\right\rangle=\mathcal{O}\left(\Delta J \frac{J^{2}}{U^{2}}\right) \tag{7}
\end{equation*}
$$

Of course, this simple model is of limited applicability for a realistic lattice in a solid-state setting, but it shows that the source term $\mathcal{M}_{\mu \nu \lambda \sigma}^{(4)}$ is nonzero, i.e., that one can create a double particle-hole pair with $\omega=2 U+\mathcal{O}(J)$.

Note that the above equation (7) is valid for small $J$ only. For larger $J$, this amplitude reaches a maximum and eventually decreases again; see Fig. 2. This double pair creation phenomenon is enabled by the interplay of hopping $J$ and interaction $U$ or, alternatively, of the correlation between sites (due to $J$ ) and the correlation between particles (due to $U$ ). Consistently, this effect vanishes both for $J=0$ [see Eq. (7)] and for $U=0$ and has maximum probability for intermediate values of $J / U$; see Fig. 2. Thus, such a signal


FIG. 2. Ratio of the probabilities for double pair creation compared to single pair creation for a tetrahedron (solid black curve) as well as fully permutationally invariant lattices with six sites (dashed blue curve) and with eight sites (dotted red curve). The top plot displays these probabilities as a function of $\arctan (J / U)$ in order to show the full range from $J / U=0$ to $J / U \rightarrow \infty$ within the finite interval $[0, \pi / 2]$. (Even though we are mostly interested in the strongly correlated Mott regime with $J \ll U$, we show full range from $J / U=0$ to $J / U \rightarrow \infty$ in order to study the overall behavior.) The bottom plot shows the same functions after the hopping rate $J$ is rescaled with the coordination number $Z$ of the lattice (i.e., the number of neighbors $v$ for a fixed lattice site $\mu$ ) via $J \rightarrow J / Z$. This suggests that the change in position and width of the peaks, when going from four to eight sites, can mainly be attributed to the increasing coordination number $Z$.
would be a signature of quantum correlations. Note that, in contrast to two-photon or multiphoton effects (Floquet theory) with the resonance condition $2 \omega=U+\mathcal{O}(J)$ or $n \omega=U+\mathcal{O}(J)$, this is a quantum effect more similar to parametric down-conversion in quantum optics (or correlated electron pair emission from single-photon absorption; see, e.g., [18]) with the resonance condition $\omega=2 U+\mathcal{O}(J)$ or $\omega=n U+\mathcal{O}(J)$.

## B. Multiple pair creation

It is also possible to create three, four, or even more pairs for $\omega=3 U+\mathcal{O}(J)$ or $\omega=4 U+\mathcal{O}(J)$, etc. However, as these processes involve higher-order correlations-e.g., for three pairs, one would have to consider the six-point correlatorthey are more and more suppressed. This suppression can be observed in Figs. 2 and 3, where the scale on the ordinate decreases by more than an order of magnitude when going


FIG. 3. Ratio of the probabilities for triple pair creation (top) and quadruple pair creation (bottom), both compared to single pair creation, for fully permutationally invariant lattices with six sites (dashed blue curve) and with eight sites (dotted red curves), again plotted as a function of $\arctan (J / U)$.
from second-order $\omega=2 U+\mathcal{O}(J)$ to third-order $\omega=3 U+$ $\mathcal{O}(J)$ and even more so for the fourth-order $\omega=4 U+\mathcal{O}(J)$.

In analogy to the tetrahedron, we considered fully permutationally invariant lattices with six and eight sites containing the same number of particles; see Fig. 1. Again restricting ourselves to the fully permutationally invariant subspace of the Hilbert space spanned by $\left|\phi_{0}\right\rangle=|\uparrow, \downarrow, \uparrow, \downarrow, \uparrow, \downarrow\rangle_{\text {symm }}$ to $\left|\phi_{3}\right\rangle=|\uparrow \downarrow, 0, \uparrow \downarrow, 0, \uparrow \downarrow, 0\rangle_{\text {symm }}$ the Hamiltonian for six sites reads

$$
\hat{H}_{\mathrm{FH}}=\left(\begin{array}{cccc}
0 & -6 J & 0 & 0  \tag{8}\\
-6 J & U-8 J & -8 J & 0 \\
0 & -8 J & 2 U-8 J & -6 J \\
0 & 0 & -6 J & 3 U
\end{array}\right)
$$

and similarly for eight sites

$$
\hat{H}_{\mathrm{FH}}=\left(\begin{array}{ccccc}
0 & -8 J & 0 & 0 & 0  \tag{9}\\
-8 J & U-12 J & -12 J & 0 & 0 \\
0 & -12 J & 2 U-16 J & -12 J & 0 \\
0 & 0 & -12 J & 3 U-12 J & -8 J \\
0 & 0 & 0 & -8 J & 4 U
\end{array}\right) .
$$

For small $J$, we find that the three-pair amplitudes $\left\langle\psi_{\text {third }}\right| \hat{H}_{\Delta J}\left|\psi_{\text {ground }}\right\rangle$ scale with $\Delta J(J / U)^{4}$ for both Hamiltonians (8) and (9), while the four-pair amplitude $\left\langle\psi_{\text {fourth }}\right| \hat{H}_{\Delta J}\left|\psi_{\text {ground }}\right\rangle$ behaves as $\Delta J(J / U)^{6}$ for the Hamiltonian (9). Again, the matrix elements vanish for $J=0$ and $U=0$ and display a single maximum at intermediate values of $J / U$; see Fig. 3 .

## III. EXPERIMENTAL REALIZATION

We now consider a possible experimental realization of multiple particle-hole pair creation as well as its spectroscopic evidence, based on femtosecond time- and angle-resolved photoemission spectroscopy (trARPES) [19,20] for which a comprehensive theoretical treatment has been developed in recent years [21-26]. In trARPES, the sample under investigation is first excited using a rather intense femtosecond optical pump pulse (see Appendix) with central frequency $\omega_{\text {pump }}$ and pulse duration $t_{\text {pump }}$. The generated nonequilibrium state is subsequently probed by means of direct photoemission using a second (weak) laser pulse at a higher frequency ( $\omega_{\text {probe }}$, $t_{\text {probe }}$ ) [27]. The overall spectral and temporal experimental resolution is then given by the convolution of both pulses and limited by the time-bandwidth product $\Delta \omega \Delta t \geqslant 4 \ln (2)$, resulting in typical values for $\Delta t$ of several tens of fs and $\hbar \Delta \omega$ of several tens of meV (Gaussian full width at half maximum). In the strongly correlated Mott regime, these conditions allow for a spectroscopic separation of the ground- and excited-state signatures (separated by the gap energy of approximately $U$ of typically a few hundred meV) but it is challenging to temporally resolve the full dynamics of individual or multiple particle-hole pairs that is expected to occur on time scales as short as $\hbar / J \approx 1 \mathrm{fs}$ [28]. Nevertheless, tracking the full dynamics is not a necessary prerequisite for the effects under discussion here and it would be a first step to observe a temporally averaged signal in the corresponding energy window.

A prototypical Mott-insulator system that has been widely investigated using trARPES (however, so far not under the conditions proposed here) is the layered transition-metal dichalcogenide $1 T-\mathrm{TaS}_{2}$ [29-32]. The Mott transition in this system goes along with the formation of commensurate charge-density wave order and a periodic lattice distortion [33], leading to a superstructure formation with rather large lattice spacing of $\ell=1.23 \mathrm{~nm}$ in a hexagonal lattice $(Z=6)$ [34]. Assuming the on-site Coulomb repulsion $U \approx 0.4 \mathrm{eV}$ and typical excitation conditions ( $E_{\text {pump }} \approx 1.4 \times 10^{8} \mathrm{~V} / \mathrm{m}$ ) reported in [29,30], the relative oscillation amplitude $\Delta J / J_{0}$ would be of the order of $20 \%$ for the second resonance $\omega=2 U+\mathcal{O}(J)$ (see Appendix). Furthermore, the reported ratio $J / U \approx 0.7$ [30] is favorable for multiple pair generation since neither $J$ nor $U$ is very small. In this particular system, signatures of doublon excitations were identified under equilibrium [35,36] and nonequilibrium conditions [37] in the spectral domain. A reasonable experimental approach to verify the generation of multiple particle-hole pairs from the absorption of single photons would thus be to verify the systematic appearance and disappearance of these excited-state signatures upon changes of the resonant pumping conditions. It should, however, be stressed that such experiments should be performed in a weak excitation limit, where only minor changes to the spectral function of the system can be expected and the pump pulse mainly acts on the individual states population.

Similar experiments might also be considered on a material class that exhibits the possibility to tune the relevant parameters. Single C, $\mathrm{Si}, \mathrm{Sn}$, or Pb adatoms on semiconductor (111) surfaces were shown to exhibit strong interelectronic Coulomb interactions, leading to energy gaps between 0.5 and 1.3 eV ( Pb and C, respectively) [38-40]. The bandwidth was found to
be significantly smaller (a few tens of meV ), but comparable for all adatom systems, allowing for studies over a relatively wide range of parameters in the deep Mott phase.

As another option for an experimental realization in a totally different range of parameters, we would like to mention ultracold atoms in optical lattices. Using fermionic atoms, the Mott insulator state has recently been realized experimentally; see, e.g., [41-43]. Since the hopping rate $J$ is directly related to the intensity of the laser forming the optical lattice, the external stimulus $J(t)$ in Eq. (2) can be realized via periodic modulations of the laser intensity. As another option, one could shake the lattice periodically with the frequency $\omega$ (see the Appendix). For optical lattices, it would even be possible to detect the number of created doublon-holon pairs via site-resolved imaging techniques; see [43]. With ultracold atoms, it is also possible to realize the bosonic version of Eq. (1), the Bose-Hubbard model, where one would expect analogous effects [44,45].

## IV. CONCLUSIONS AND OUTLOOK

We studied the Fermi-Hubbard model (1) deep in the strongly correlated Mott phase under the influence of an oscillating hopping rate (2). In addition to the well-known fundamental resonance $\omega=U+\mathcal{O}(J)$, we find higher resonances at $\omega=2 U+\mathcal{O}(J)$ and $3 U+\mathcal{O}(J)$ and so on, which correspond to the creation of multiple particle-hole pairs. This multiple pair creation effect is caused by the interplay between the correlations between particles (due to $U$ ) on the one hand and the correlations between lattice sites (due to $J$ ) on the other hand. Thus it is a genuine signature of these nontrivial correlations.

Consistent with this picture, we found that triple pair creation $\omega=3 U+\mathcal{O}(J)$ is more suppressed than double pair creation $\omega=2 U+\mathcal{O}(J)$ while the creation of four pairs $\omega=4 U+\mathcal{O}(J)$ is even more suppressed; see Figs. 2 and 3. We attribute this behavior to the hierarchy of correlations, i.e., the fact that-at least in the absence of symmetry breaking-the correlations between three lattices sites are typically smaller than the correlations between two sites and so on; see, e.g., [9,10,15-17]. However, further investigations are necessary to understand this effect better [44].

We also discussed experimental parameters which show that this effect should be observable in time domain experiments investigating solid states or ultracold atoms in optical lattices. Note that the multiple pair creation effect considered here is different from charge carrier multiplication such as impact ionization; see, e.g., [49], where an excitation generates further particle-hole pairs after its creation. In contrast, the effect considered here describes the generation of multiple pairs simultaneously by one and the same pump photon.

Interestingly, for the Fermi-Hubbard model on a square (instead of a tetrahedron; cf. Fig. 1), we do not find this multiple pair-creation effect-at least not in the fully symmetric subspace. Whether this is a result of these symmetries, the reduced coordination number $Z$ (two instead of three), or the bipartite structure of the square which facilitates antiferromagnetic Néel ordering of the spins should be clarified in future investigations. Note that, after taking the Coulomb interactions between different lattice sites into account (in the extended

Fermi-Hubbard model), we get double pair creation (similar to Auger processes) also on a square [44].

In summary, we find that the dynamics of strongly correlated quantum many-body systems is still not fully understood and can afford surprises, which motivates further studies.

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## APPENDIX: PUMP PULSE

Let us briefly discuss the impact of a pump laser on the Fermi-Hubbard model (1) and show how it corresponds to the oscillating hopping rate (2). For simplicity, we consider the Fermi-Hubbard model in one dimension:

$$
\begin{equation*}
\hat{H}_{\mathrm{FH}}^{\mathrm{D}}=-J \sum_{\mu, s}\left(\hat{c}_{\mu, s}^{\dagger} \hat{c}_{\mu+1, s}+\text { H.c. }\right)+U \sum_{\mu} \hat{n}_{\mu}^{\dagger} \hat{n}_{\mu}^{\downarrow} . \tag{A1}
\end{equation*}
$$

In higher dimensions, additional factors can play a role (such as the direction of hopping relative to the polarization of the pump laser) but the main effect remains the same. Neglecting the magnetic component of the pump laser, the most obvious impact of the pump field (others will be discussed below) is a time-dependent shift of the energies corresponding to the Hamiltonian

$$
\begin{equation*}
\hat{H}_{\mathrm{pump}}(t)=\sum_{\mu}\left(\hat{n}_{\mu}^{\hat{1}}+\hat{n}_{\mu}^{\downarrow}\right) V_{\mu}(t) . \tag{A2}
\end{equation*}
$$

Such Hamiltonians are often discussed in the context of driven quantum lattice systems. Assuming that the typical laser wave numbers $k_{\text {laser }}^{\|}$parallel to the lattice are small compared to the other relevant scales, the site-dependent energy shift $V_{\mu}(t) \approx$ $-q \mathbf{r}_{\mu} \cdot \mathbf{E}(t)$ at the position $\mathbf{r}_{\mu}$ of the site $\mu$ is determined by the electric pump field $\mathbf{E}(t)$ with $q$ being the elementary charge. This Hamiltonian (A2) generates the Peierls transformation

$$
\begin{equation*}
\hat{c}_{\mu, s}(t) \rightarrow \hat{c}_{\mu, s}(t) e^{i \varphi_{\mu}(t)}, \tag{A3}
\end{equation*}
$$

with the time-dependent phase $\dot{\varphi}_{\mu}(t)=V_{\mu}(t)$. Inserting this transformation back into Eq. (1), we find that the tunneling term $\propto J$ acquires an oscillating phase

$$
\begin{equation*}
J \rightarrow J(t)=J_{0} e^{i \Delta \varphi(t)} \tag{A4}
\end{equation*}
$$

where $\Delta \varphi(t)$ denotes the relative phase difference between neighboring lattice sites.

Assuming a harmonically oscillating time dependence, we may insert $E^{\|}(t)=E^{\|} \cos (\omega t)$ and obtain

$$
\begin{equation*}
\Delta \varphi(t)=q \ell E^{\|} \frac{\sin (\omega t)}{\omega}=\Delta \varphi_{\max } \sin (\omega t) \tag{A5}
\end{equation*}
$$

with the lattice spacing $\ell$.

## 1. Effective quantum quench

If the pump frequency $\omega$ is much larger than all the other relevant energy scales such as $J$ and $U$, the main consequence of the time dependence $[\mathrm{Eq}$. (A4)] is that the original hopping
rate $J$ in the Hamiltonian (A1) can effectively be replaced by the time-averaged hopping rate $\bar{J}$. For a harmonic oscillation, we may calculate the time average via the Jacobi-Anger expansion and obtain

$$
\begin{equation*}
\bar{J}=\overline{J_{0} e^{i \Delta \varphi(t)}}=J_{0} \tilde{J}_{0}\left(\Delta \varphi_{\max }\right) \tag{A6}
\end{equation*}
$$

where $\mathfrak{J}_{0}$ denotes the Bessel function of the first kind. Since $\left|\mathfrak{J}_{0}\right| \leqslant 1$, the effective time-averaged hopping rate is lowered by the pump beam. For certain values of $\Delta \varphi_{\max }$ such as $\Delta \varphi_{\max }^{0} \approx 2.4$, one may even effectively inhibit hopping due to $\mathfrak{J}_{0}\left(\Delta \varphi_{\max }^{0}\right)=0$. Thus, if we would switch on (or off) the pump beam sufficiently quickly, i.e., faster than the characteristic response time of our system, the situation would be very analogous to a quantum quench as discussed in [3,4,6-10], for example. As shown in these papers, such a quench will create particle-hole (doublon-holon) pairs in generalthe number (density) of those pairs will depend on the parameters such as $U$ and the initial $J_{\text {in }}$ and final $J_{\text {out }}$ hopping rates.

If the pump field $E$ is weak enough such that the phase $\Delta \varphi_{\max } \ll 1$ is small, a Taylor expansion gives

$$
\begin{equation*}
\bar{J} \approx J_{0}\left(1-\frac{1}{2} \overline{\Delta \varphi^{2}(t)}\right)=J_{0}\left(1-\frac{1}{4} \Delta \varphi_{\max }^{2}\right) . \tag{A7}
\end{equation*}
$$

In this case, the change of the hopping rate is relatively small $\Delta \bar{J}=-J_{0} \Delta \varphi_{\max }^{2} / 4$ and thus we may employ time-dependent perturbation theory where the perturbation Hamiltonian is governed by $\Delta \bar{J}$. As the perturbation Hamiltonian scales quadratically in $\Delta \varphi \ll 1$ and thus linearly in the pump intensity $I_{\text {pump }} \propto E_{\text {pump }}^{2}$, the probability for pair creation (per unit length) would be suppressed as the fourth power of $\Delta \varphi \ll 1$, i.e., it would scale quadratically in the pump intensity $P \propto E^{4} \propto I^{2}$. This scaling could help to distinguish the above quench mechanism from other effects such as linear dipoletype transitions (which scale linearly in $I$, for example). On the other hand, the scaling indicates that this is a second-order effect, which is typically suppressed in comparison to potential competing first-order effects.

## 2. Resonant excitations

Such first-order effects arise when the pump frequency $\omega$ is not much larger than all other energy scales, which is the scenario considered in this paper. In this case, a Taylor expansion of Eq. (A4) reproduces Eq. (2) to first order.

One should also keep in mind that the Hamiltonian (A2) only contains the component of the electric field parallel to the lattice-while the perpendicular component can also induce effects such as the deformation of wave functions leading to variations of $J$ and $U$, i.e., it can also cause small oscillations in $J$ and $U$. However, assuming that the initial state is the ground state (i.e., an eigenstate) of the Hamiltonian (A1), the perturbation caused by a small variation of $U(t)$ is equivalent (to lowest order) to the perturbation caused by an appropriate small variation of $J(t)$.

In higher-dimensional lattices, $\Delta J(t)$ can also depend on the lattice indices $\Delta J_{\mu \nu}(t)$, e.g., on the direction relative to the pump beam, but we omit this dependence for simplicity here. More generally, repeating the steps of the derivation of the Fermi-Hubbard Hamiltonian (1) from the underlying
many-body Hamiltonian (including the Coulomb interaction) in the presence of the pump field, one would also obtain
oscillating terms like $\hat{c}_{\mu, s}^{\dagger} \hat{c}_{\nu, s^{\prime}}^{\dagger} W_{\mu \nu \lambda \sigma}^{s s^{\prime}}(t) \hat{c}_{\lambda, s} \hat{c}_{\sigma, s^{\prime}}$, but we do not consider these contributions here.
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